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Math√oot

Essential for Every Science

E-Magazine

Alva's Institute of Engineering and Technology, Moodbidri

Department of Mathematics

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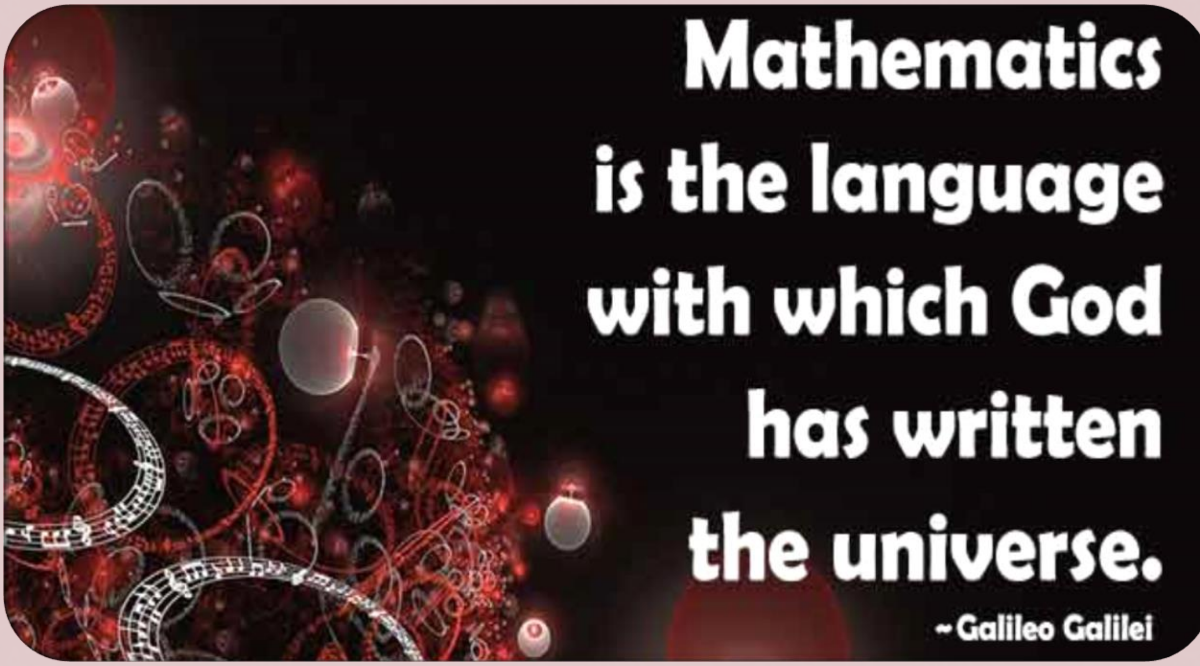


From Chief Editor's Desk:

Dear Reader,

True words by a famous astronomer Galileo Galilei. Every single structure in nature involves mathematics, whether it be a honeycomb, a flower's petal arrangement, a spider web, or a geographical pattern. Mathematics is now employed as a vital tool in many sectors around the world, including natural science, engineering, medical, and the social sciences. Every real life problem when explored, develops into a Mathematical problem and can be solved with accuracy.

Mathroot is indeed an e–magazine published by the Department of Mathematics with the goal of raising awareness about the use of mathematics in various fields. This magazine will also include some interesting facts about Mathematics and Mathematicians, which will inspire the reader. Every AITEIAN can contribute to the magazine in their own way and make it more attractive and meaningful. Thank you.

- Dr. Prameela Kolake

Dept. of Mathematics

FOURIER SERIES

By: Laxmish Vishnu Hegde
4AL20CS064

In mathematics, Fourier series is a periodic function composed of harmonically related sinusoids combined by a weighted summation. With appropriate weights, one cycle (or period) of the summation can be made to approximate an arbitrary function in that interval. As such, the summation is synthesis of another function. The discrete-time Fourier transform is an example of Fourier series. The process of deriving weights that describes the given function is a form of Fourier analysis. For functions on unbounded intervals, the analysis and synthesis analogies are Fourier transform and inverse transform.

APPLICATION OF FOURIER SERIES:

- Signal Processing – It is the best application of Fourier analysis.
- Approximation Theory – The Fourier series is used to write the given function as a trigonometric polynomial.
- Control Theory – The Fourier series of functions in the differential equation often gives some prediction about the behavior of the solution of differential equation.
- Partial Differential equation – Fourier analysis is used to solve higher order partial differential equations by the method of separation of variables.

APPLICATION OF FOURIER SERIES IN MECHANICHAL ENGINEERING:

The most useful application is in solving vibration problems. Generally, vibrations are difficult to model and have to solve it using numerical techniques such as a Fourier Series coupled with the Method of Multiple Scales or any number of other advanced technique.

Although the original motivation was to solve the heat equation, it later became obvious that the same techniques could be applied to a wide array of mathematical and physical problems.

Fourier series is applicable to any phenomenon which involves frequency, that is, any complex phenomenon involving a frequency spectrum can be solved using FOURIER ANALYSIS.

Demodulation

Demodulation is extracting the original information-bearing signal from a carrier wave. A demodulator is an electronic circuit (or computer program in a software-defined radio) that is used to recover the information content from the modulated carrier wave. There are many types of modulation so there are many types of demodulators. The signal output from a demodulator may represent sound (an analog audio signal), images (an analog video signal) or binary data (a digital signal).

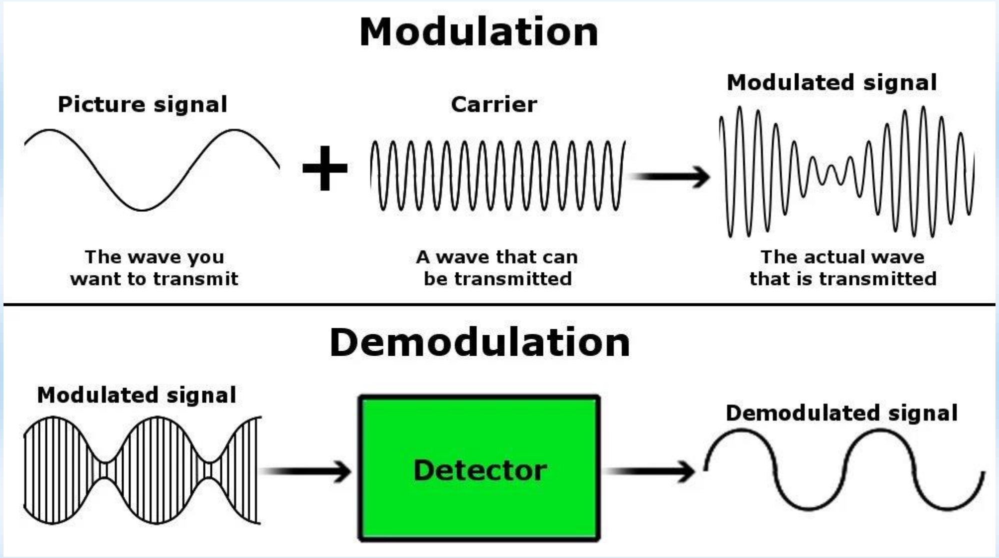
Modulation

In electronics and telecommunications, modulation is the process of varying one or more properties of a periodic waveform, called the carrier signal, with a separate signal called the modulation signal that typically contains information to be transmitted.

Difference Between Modulation and Demodulation

Modulation is the process of influencing data information on the carrier, while demodulation is the recovery of original information at the distant end of the carrier.

A modem is an equipment that performs both modulation and demodulation. Both processes aim to achieve transfer information with the minimum distortion, minimum loss, and efficient utilization of spectrum. Even though there are different methods for modulation and demodulation processes, each has its own advantages and disadvantages. For example, AM is used in shortwave and radio wave broadcasting; FM is mostly used in high-frequency radio broadcasting, and pulse modulation is known for digital signal modulation.



APPLICATION OF ANALYTIC FUNCTIONS

By: LIKHITH CG
4AL20CS065

Electrostatic Fields:

Surface Electrode Ion Trap:

- The design of surface electrode ion traps is studied analytically.
- The classical motion of a single ion in such a trap is described by Coupled Mathieu equations, analytic boundary value solution using Laplace equation.
- Also, this solution is used to model the electrostatic potential field generated which helps in calculating important trap design parameters such as center position of the trapped ion, trap depth and stability parameters.

- The model is used to determine optimized dimensions for trap electrodes in various case

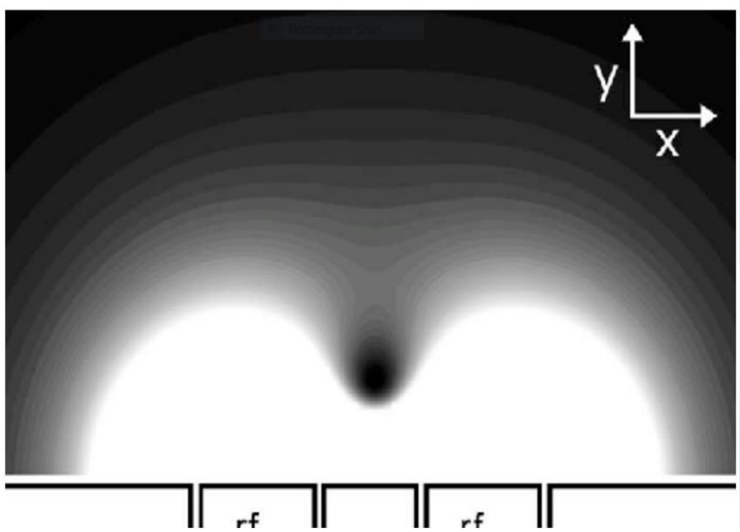
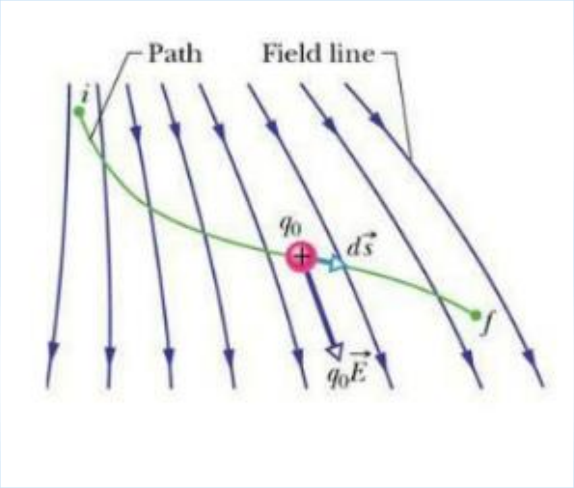


Illustration of the pseudopotential well created by the electric fields of the surface electrodes, with the electrodes diagrammed at bottom. Dark shades correspond to low pseudopotential energy. The ion is to be trapped in the potential well near the center of the figure. The lowest energy path for the ion to escape the trap is through the saddle point in the pseudopotential directly above; the difference in pseudopotential between the center of the well and this escape point represents the maximum energy the ion can have and remain trapped. The top surface of the electrodes represent the y = 0 plane. The electrodes labeled "rf" have the oscillating voltage applied to them; the others are rf grounded.

Electrostatic Potential on Different Surfaces:

- The Concept of Potential in Physics can simplify the derivation of forces.
- So, these potentials are typically described by using these analytic



functions.

Incompressible Fluid Flow Analysis:

- In this Fluid flow analysis, we run fluid simulation to measure certain parameters such as viscosity, temperature, velocity, pressure etc…
- SIM SCALE is one of the fluid simulation software to find the required parameters.
- Software follows some constraints before measuring the parameters.
- Initialization of the simulation can be done by giving CAD model as input.

RUNGE-KUTTA METHOD

By: Hardik Prabhu
4AL20CS045

Introduction:

The Runge-Kutta Method was developed by two German men Carl Runge (1856-1927), and Martin Kutta (1867- 1944) in 1901. Carl Runge developed numerical methods for solving the differential equations that arose in his study of atomic spectra. These numerical methods are still used today.Runge–Kutta method is an effective and widely used method for solving the initial-value problems of differential equations. Runge–Kutta method can be used to construct high order accurate numerical method by function's self without needing the high order derivatives of functions.

Runge-Kutta methods are a family of iterative methods, used to approximate solutions of Ordinary Differential Equations (ODEs). Such methods use discretization to calculate the solutions in small steps. The approximation of the "next step" is calculated from the previous one, by adding s terms

Formula for RK method

RK4 is an iterative method to find out the approximate solution of ODE (Ordinary Differential Equation). Starting with an initial given condition we calculate forward step by step using RK4 algorithm. The differential equation with an initial condition are given. We need to find solution for some. Here is the number of iterations. Now we define the step size with the above values, the RK4 method is as follows, here is an unknown function of x for which we have to find approximate solution.

Runge-Kutta method (actually due to Kutta). If the differential equation is,

$$\frac{dy}{dx}=f(x, y),$$

The step from Xn to xn+l=xn+h is made by the formulas,

$$y_{n+1}=y_n+(k_1+2k_2+2k_3+k_4)/6,$$

$$k_1=hf(x_n, y_n),$$
$$k_2=hf(x_n+\frac{1}{2}h, y_n+\frac{1}{2}k_1),$$
$$k_3=hf(x_n+\frac{1}{2}h, y_n+\frac{1}{2}k_2),$$
$$k_4=hf(x_n+h, y_n+k_3).$$

These formulae yield an approximation of the fourth degree in h, that is, the expansion in powers of h defined by these formulas agrees through terms of the fourth degree with the expansion obtained directly from the differential equation.

Applications of RK method

Experimental Analysis of Low Earth Orbit Satellites due to Atmospheric Perturbations

A satellite is expected to move in the orbit until its life is over. This would have been true if the earth was a true sphere and gravity was the only force acting on the satellite. However, a satellite is deviated from its normal path due to several forces. This deviation is termed as orbital perturbation. The perturbation can be generated due to many known and unknown sources such as Sun and Moon, Solar pressure, etc. This paper discusses the study of perturbation of a Low Earth satellite orbit due to the presence of aerodynamic drag. Numerical method (Runge – Kutta fourth order) is used to solve the Cowell's equation of perturbation, which consists of ordinary differential equation.

Computation of Acceleration of a Rocket

Rocket is a long circular device that is launched into the air. An example of a rocket is what helps launch a guided missile into space. An example of a rocket is a firecracker. There are several stages in the Rocket.

Staging is the combination of several rocket sections, or stages, that fire in a specific order and then detach, so a ship can penetrate Earth's atmosphere and reach space. The operative principle behind rocket stages is that you need a certain amount of thrust to get above the atmosphere, and then further thrust to accelerate to a speed fast enough to stay in orbit around Earth (orbital speed, about five miles per second).It's easier for a rocket to get to that orbital speed without having to carry the excess weight of empty propellant tanks and early-stage rockets. So, when the fuel/oxygen for each stage of a rocket is used up, the ship jettisons that stage, and it falls back to Earth. This becomes part of the rocket's mass fraction—the portion of its fully fueled pre-launch mass that does reach orbit.

Let's consider a rocket that is flying through the Earth's atmosphere. First of all, we have the ODE to calculate the acceleration:

Acceleration= (rocket force + force drag) / mass

We know that acceleration is the derivative of velocity (as mentioned earlier) so using RK4 to calculate this should be relatively straight forward

K1 = ODE (time + velocity)

K2 = ODE (time + 1/2 timeElapsed, x + 1/2 * K1 * timeElapsed)

K3=ODE (time + 1/2 * timeElapsed, x + 1/2* K2 timeElapsed)

K4 = ODE (time + timeElapsed, x + K3 timeElapsed)

And finally, to calculate the acceleration:

Acceleration n+1 = Acceleration n + 1/6 (K1+2* K2 +2* K3 +K4) * time Elapsed

So, in this way we calculate the acceleration of rocket in different stages to get the brief approximation of fuel abundancy and speed of the rocket.

Flight simulation of a Rocket

Flight simulator is a software that can simulate rocket flight allows for numerous hours to be saved during the design process. Additionally, the option to change certain variables and parameters to simulate different models awards the user with the option to experiment with different ideas and design philosophies. But how would someone with little knowledge of designing a rocket be able to perform such a task? This Program that allows a user with little knowledge the same ability to predict the flight path of a conceptually designed rocket as those with more experience. Here in this Rocket simulation the Runge-kutta Method is being used to predict the flight path along with other calculations. The part of code given below uses RK method to perform the calculation in a flight simulator program,

A.3.2 4th Order Runge-Kutta Methods

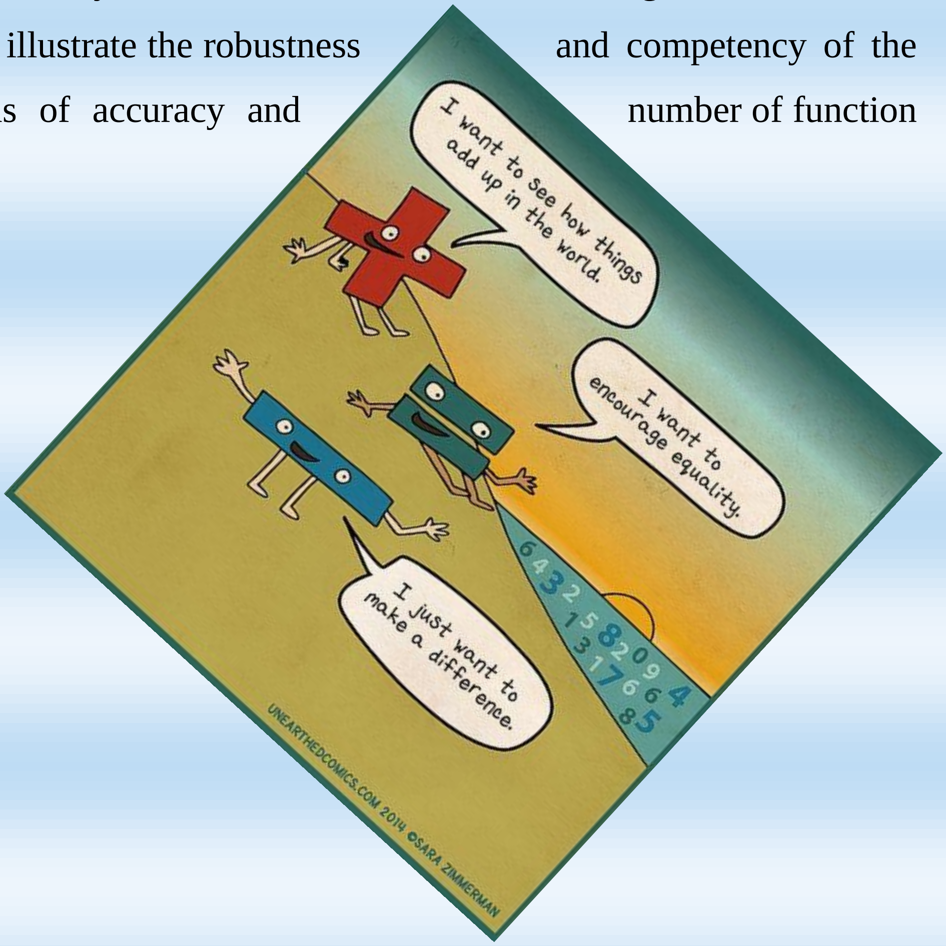
A.3.2.1 4th Order X-Y

```
clc; clear all
global thetao dt Co_Drag

%% Call Initial Conditions
Initial_Conditions_Compiler

%% Set Addition Parameters
dt = TimeStep;
n = (tbo(end)*100);
tspan = [0:dt:n];
thetao = theta(1);
if DCondition == 3
    Co_Drag = input('Enter Constant Drag Value. ');
end

% Set Initial Values for ode function
xo = 0;
yo = 0;
mo = m(1);
uo = u(1);
vo = v(1);
y0 = [uo vo xo yo mo];

% Set the limit for the ode function
Opt = odeset('Events', @StopFunc);

[t,y] = ode45(@(t,y) ode_func_x_y(t,y),tspan,y0,Opt);
```

Here ODE45 is MATLAB's standard solver for ordinary differential equations (ODEs) is the function. This function implements a Runge-Kutta method with a variable time step for. efficient computation.

Conclusion

A Runge-Kutta type method for directly solving special fourth-order ordinary differential equations (ODEs) which is denoted by RKFD method is constructed. The order conditions of RKFD method up to order five are derived; based on the order conditions, three-stage fourth- and fifth-order Runge-Kutta type methods are constructed. Zero-stability of the RKFD method is proven. Numerical results obtained are compared with the existing Runge-Kutta methods in the scientific literature after reducing the problems into a system of first-order ODEs and solving them. Numerical results are presented to illustrate the robustness and competency of the new methods in terms of accuracy and number of function evaluations.



MATHEMATICS IN NATURE

By: ARUNDHATHI S BHAT
4AL19IS008

Mathematics is one of the most fundamental of all the sciences governing our universe. Imagine our own world without it. If we didn't know about addition and subtraction and the various calculations, none of the worlds would have been the way it is. It the mathematics that has made mankind learn so much about the universe, ocean depths, earthquake intensities etc. Here an important question arises; is math essentially a part of nature or was it developed to actually understand and interpret nature?

Diverse opinions exist in this regard. Some believe math was developed by man intentionally with the purpose of understanding this giant network laid by God. The reason for them to think this way is if math was part of nature, then man would have been born with a natural understanding for it but man instead, has to take classes and think and learn math.

Though math does help us in measuring the out and about nature, yet it is one of the most unspecific ways of actually measuring nature. This is because other sciences like physics and chemistry measure and study about the actual components of our universe. Physics can tell us which force is acting where and why. Chemistry can tell us about the various chemical reactions occurring throughout our universe and their consequences. Math, on the other hand, is merely a number game. It only helps to measure and estimate. Hence it is a generic science applicable to all other sciences and that's what the magic is math is.

When the words 'mathematics' and 'nature' are put together in the same sentence, amongst the first things that comes to mind are the Fibonacci sequence and The Golden Ratio, two of the most mundane examples where mathematics and nature seem to entwine. But are we really looking at examples where mathematics decrees the way in which nature acts, hence being at the "heart" of it? I personally think not, simply because the notion where an abstract man-made concept plans out a world far older than life itself seems at the very least controversial.

This does not go to say that mathematics is as a whole, completely irrelevant in the natural world, but the number of situations where through mathematics only an accurate and valid prediction can be made are both few and incoherent. The fact that mathematics can be seen as being at the heart of nature stems from the fact that the mathematics systems we practice and believe in today have been empirically proven. But what were to happen if someone were able to show that is impossible to prove that a formal mathematical system is free from contradiction?

This is precisely what Kurt Gödel did in 1931, according to his theorem we cannot be certain that mathematics does not contain contradictions. Assuming mathematics was at the heart of nature, this theorem would mean that there could be a fundamental contradiction in nature. Expanding upon this further; Einstein once said, "As far as the laws on mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. Effectively, contradictions could not exist in nature, therefore proving that mathematics could not "be at the heart of nature" for then it would surreal.

A statement that is probably more fitting would be "mathematics is man's attempt at reflecting and understanding nature". However, this is not entirely precise either, because there are many things mathematics can do which go beyond the scope of nature such as algebra, calculus, or even simple multiplication. All these things are methods by which mathematicians try to construct models, which are isomorphic, or show homomorphism at the very least to the vast universe of nature. One of the most famous, and seemingly pointless examples is Buffon's needle problem, posed in 1733.

The problem consisted in finding the probability that a needle of length X dropped at random onto a piece of paper with equally spaced parallel lines X distance apart (same length as needle). The answer to the problem surprisingly turns out to be 2/. Why should come up is still a complete mystery and raises interesting questions about the 'the unreasonable effectiveness of mathematics'. Among other things, has been around for c. 4000 years, and could be considered the isomorphic epitome of man's goal when it comes to understanding nature.

Apart from its mysterious appearance in Buffon's needle problem? is the most fundamental figure when doing any circle-related calculations – be it the area, circumference, radii, or chords. One can argue that the figure alone is proof that mathematics is at the heart of nature, however if we look closer at what? really is, as a number irrational (can't be written in the format x/y), and is therefore endless – it becomes obvious that it is not part of nature, but simply trying to mirror it. The fact that it does so, so well shows that it is near isomorphic in its perfection.

CONCLUSION:

I think that our mathematics at best, can be identical to a small part of nature – homomorphic to nature. Which basically means that all our mathematics can in one way or another be related to a natural element, on the other hand we will never know if the homomorphism we take for isomorphism is what it actually is, or if we were wrong all along and it is not. Additionally, we will never know if we will be able to relate every natural recombination with a mathematical equation.

INFINITY AND ITS FACTS

By: Aparna Nath A.S

First year (Data Science 2022-23 batch)

Mathematics is the science of numbers and shapes. It is considered as an interdisciplinary language biologists, linguists and sociologists alike use math in their work. Mathematics and mathematical terms are part of our day to day life . In that context when we see, in our daily conversations we often use the word 'infinity'. What is infinity? How did it evolve ? And what does that mean in mathematics and literature, these are the usual questions that haunt us.

Infinity is a number greater than any assignable quantity or countable number . It is a big topic. Most people have some conceptions of things that have no bound, no boundary, no limit, no end. The rigorous study of Infinity began in mathematics and philosophy but the engagement with Infinity traverses the history of cosmology, astronomy, physics and theologus. Mathematics itself has appealed to some form of Infinity from its beginning and its contemporary practice requires infinitely foundations. Any field that employs mathematics at least flirts with infinity indirectly and in many cases courts it directly.

Potential infinite occurs in Greek mathematics from the outset, most obviously in the nature number series and in the geometrical operations of addition and division of segments and other geometrical magnitudes. The Greek mathematicians, starting with Eudoxus, developed a technique for measuring plain and solid figures that avoided recovers to the infinite even where an infinite "limit" process would seem to be forced by the situation. This technique today is known as method of exhaustion. Greek mathematics generally avoids any recourse to the actual infinite and scholars have spoken of a 'horror of Infinity' typical of Greek mathematics.

Infinity made its appearance in 17th century geometry in the work of Desargues where is in Euclidean geometry parallel lines do not meet, Desargues entertained the idea of having parallel lines meet at a point at Infinity. This was a very fruitful idea that led to the development of projective geometry. The most fruitful development in the use of Infinity in 17th century mathematics was that of calculus. The problem of extending the concept of counting from the finite to the Infinity. This problem is related to the issue of whether there is only one Infinity or whether there might be different sizes of Infinity. Mathematics and mathematical terms are part of our daily life. Being familiar with them will make us comfortable to manage things easier. Infinity is such a beautiful word that we use often, its evolution and history makes us more interesting to know more about the topic.

TAYLOR'S SERIES

By: Dr. Prameela Kolake

History:

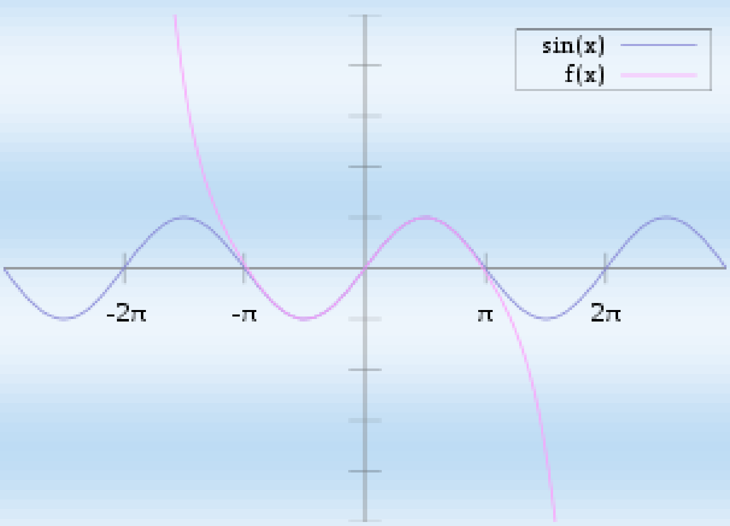
The Greek philosopher Zeno considered the problem of summing an infinite series to achieve a finite result, but rejected it as an impossibility. The result was called as Zeno's paradox. Later, Aristotle proposed a philosophical resolution of the paradox, but the mathematical content was apparently unresolved until taken up by Archimedes, as it had been prior to Aristotle by the Presocratic Atomist Democritus. It was through Archimedes' method of exhaustion that an infinite number of progressive subdivisions could be performed to achieve a finite result. Liu Hui independently employed a similar method a few centuries later.

In the 14th century, the earliest examples of the use of Taylor series and closely related methods were given by Madhava of Sangamagrama. Though no record of his work survives, writings of later Indian mathematicians suggest that he found a number of special cases of the Taylor series, including those for the trigonometric functions of sine, cosine, tangent, and arctangent. The Kerala School of Astronomy and Mathematics further expanded his works with various series expansions and rational approximations until the 16th century.

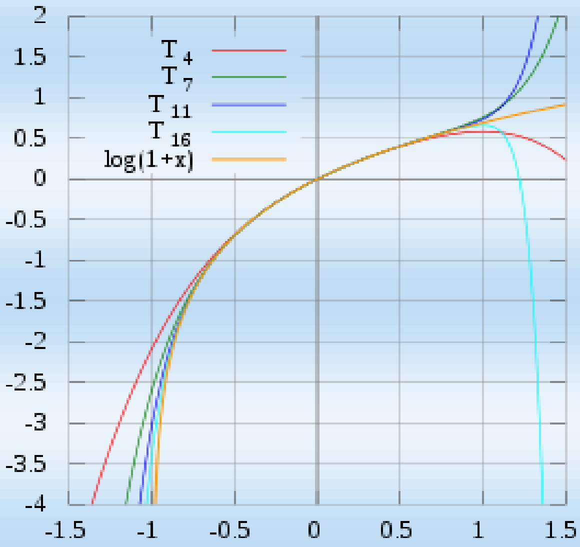
In the 17th century, James Gregory also worked in this area and published several Maclaurin series. It was not until 1715 however that a general method for constructing these series for all functions for which they exist was finally provided by Brook Taylor,[7] after whom the series are now named.

The Maclaurin series was named after Colin Maclaurin, a professor in Edinburgh, who published the special case of the Taylor result in the 18th century.

Geometrical comparison of Taylor's Series of some function:



The sine function (blue) is closely approximated by its Taylor polynomial of degree 7 (pink) for a full period centered at the origin. Pictured on the left is an accurate approximation of sin x around the point $x = 0$. The pink curve is a polynomial of degree seven.



The Taylor polynomials for ln(1+x) only provide accurate approximations in the range $-1 < x \leq 1$. For $x > 1$, Taylor polynomials of higher degree provide worse approximations.

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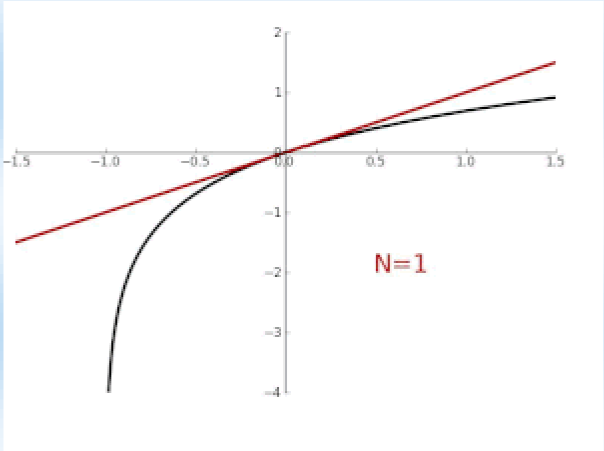
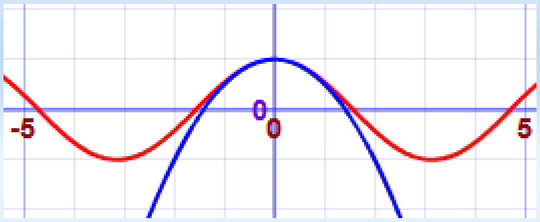


The Taylor approximations for ln(1+ x) (black). For $x > 1$, the approximations diverge.

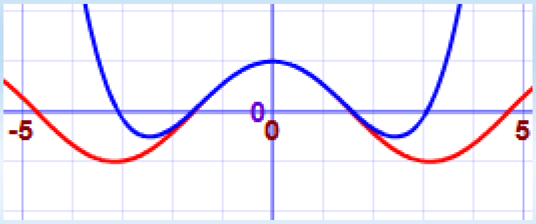
We can use the first few terms of a Taylor Series to get an approximate value for a function.

Here we show better and better approximations for **cos(x)**. The red line is **cos(x)**, the blue is the approximation

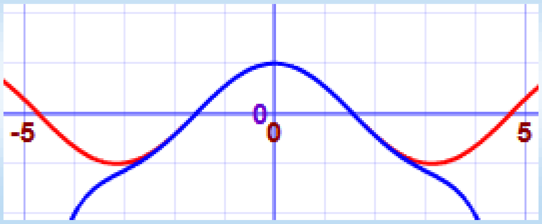
$\text{Cos}(x) = 1 - x^2/2!$



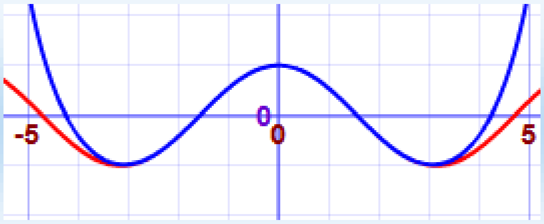
$\text{Cos}(x) = 1 - x^2/2! + x^4/4!$



$\text{Cos}(x) = 1 - x^2/2! + x^4/4! - x^6/6!$



$\text{Cos}(x) = 1 - x^2/2! + x^4/4! - x^6/6! + x^8/8!$



Application of Taylor's series:

Taylor Series are studied because polynomial functions are easy and if one could find a way to represent complicated functions as series (infinite polynomials) then one can easily study the properties of difficult functions.

- Evaluating definite Integrals: Some functions have no antiderivative which can be expressed in terms of familiar functions. This makes evaluating definite integrals of these functions difficult because the Fundamental

Theorem of Calculus cannot be used. If we have a polynomial representation of a function, we can oftentimes use that to evaluate a definite integral.

- Understanding asymptotic behavior: Sometimes, a Taylor series can tell us useful information about how a function behaves in an important part of its domain.
- In the calculator era, we often don't realize how deeply nontrivial it is to get an arbitrarily good approximation for a number like e , $e^{sin\sqrt{2}}$. Since it's analytic, i.e. has a Taylor series, if we want to compute its values we just compute the first few terms of its Taylor expansion at some point.
- We use generating functions solve discrete counting problems to the continuous, where things are easy. Generating functions are a central tool in combinatorics (counting, graph theory, etc.) and probability (where we have moment generating functions). Taylor series is the fundamental idea behind all of these.
- A good example of Taylor series and, in particular, the Maclaurin series, is in special relativity, where the Maclaurin series are used to approximate the Lorrentz factor γ. Taking the first two terms of the series gives a very good approximation for low speeds. In physics you often approximate a complicated function by taking the first few terms in the Taylor series (the Taylor polynomial). For small values of the independent variable, you often assume linearity, which can allow you to get a closed form solution. For example, if you take an introductory physics class then you usually study the motion of the pendulum by approximating $sin(\theta)$ by θ for small angles.
- Moreover, any software that graphs various functions actually uses very good Taylor approximations.

Dear reader, above are the glimpses of application of Taylor's series expansion. Plenty of such applications can be found in books, web pages and elsewhere. Go ahead and explore more on Taylor's series.....

DEPARTMENT ACTIVITIES

National Mathematics Day – 2021

The Birth anniversary of renowned Indian Mathematician Srinivasa Ramanujan, December 22, was announced as National Mathematics Day by then Prime Minister Dr. Manmohan Singh on the 125th birthday of Srinivas Ramanujan at Madras University. Henceforth, various educational institutions throughout the nation are celebrating the day with great enthusiasm. With great spirit, the Department of Mathematics, AIET, Mijar celebrated National Mathematics Day 2021 on December 30, 2021, at the Auditorium of AIET.

On behalf of National Mathematics Day, mathematics puzzle-solving and science model competitions were held for students. The puzzle-solving competition was conducted on December 23, 2021 at 3.30 PM in Rooms No. 403 and 412 of the Main Block. 87 students from first year to final year actively participated in this

competition. The Science Model Competition was held on the day of celebration, i.e., December 30, 2021.

Mr. Yakoob Koyyur, Head Master of GHS Nada, Belthangady, Chief guest for the programme, Dr. Shashi Kumar from the Physics Dept., and Dr. Ajith Hebbar from the Civil Engg. Dept., judged the competition. The principal, faculty, and students from various branches witnessed the program. Soon after the competition, the formal function started. Nithin Hemaraj hosted the program. The program started with a prayer by Sharanya and Arundhathi from 5^{th} semester ISE. Then Dr. Prashanthi K. S. welcomed the dignitaries and guests and introduced the Chief Guest Mr. Yakub Koyyur. Sathyam Pawale from 3rd-semester AIML announced the prize winners' names and the Chief Guest handed over the prize.

The winners of the mathematics puzzle-solving competition were Nithin Hemaraj from 3rd-semester of AIML secured 1st place and 2nd place was secured by two students, Sayeed A.R. from CSE 3rd semester and Ullas H.U. from AIML 3rd semester. K K Kaushik and team from 3rd semester CSE won first place, Satyam Pawale and team from 3rd semester AIML got 2nd place, and Fathima Thashiba and team from 3rd semester received 3rd place in the Science Model Making Competition. Then Dr. Prameela Kolake, HoD of the Mathematics Department, gave preliminary speech, which was followed by the launch of the e-magazine "Mathroot" by the dignitaries. Mathroot contains content and concepts about mathematics for learners.

After the launch of the e-magazine, Chief Guest addressed the gathering and, in his address, Mr. Yakub Koyyur, beautifully narrated Srinivasa Ranujan's life story and also advised the students about the importance of studies in life by giving examples of great souls like Dr. A P J Abdul Kalam, Sir M Visweshwarayya. The principal then addressed and encouraged the gathering to take mathematics not as a subject and treat it as a source of enjoyment and development. At the end, Keerthana from 3rd-semester AIML delivered a vote of thanks. As Twilight drops her curtain down and pins it with a star, the programme ended with the wonderful memories of the day.